

BEST POST-TRANSFORM SELECTION IN A RATE-DISTORTION SENSE

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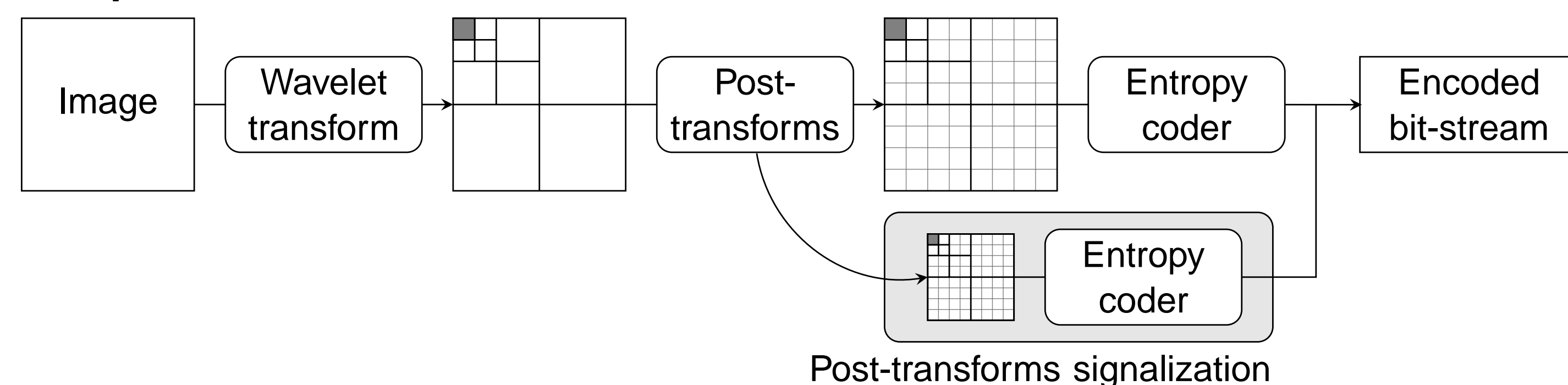
PROBLEM STATEMENT

The Post-transform Compression Scheme:

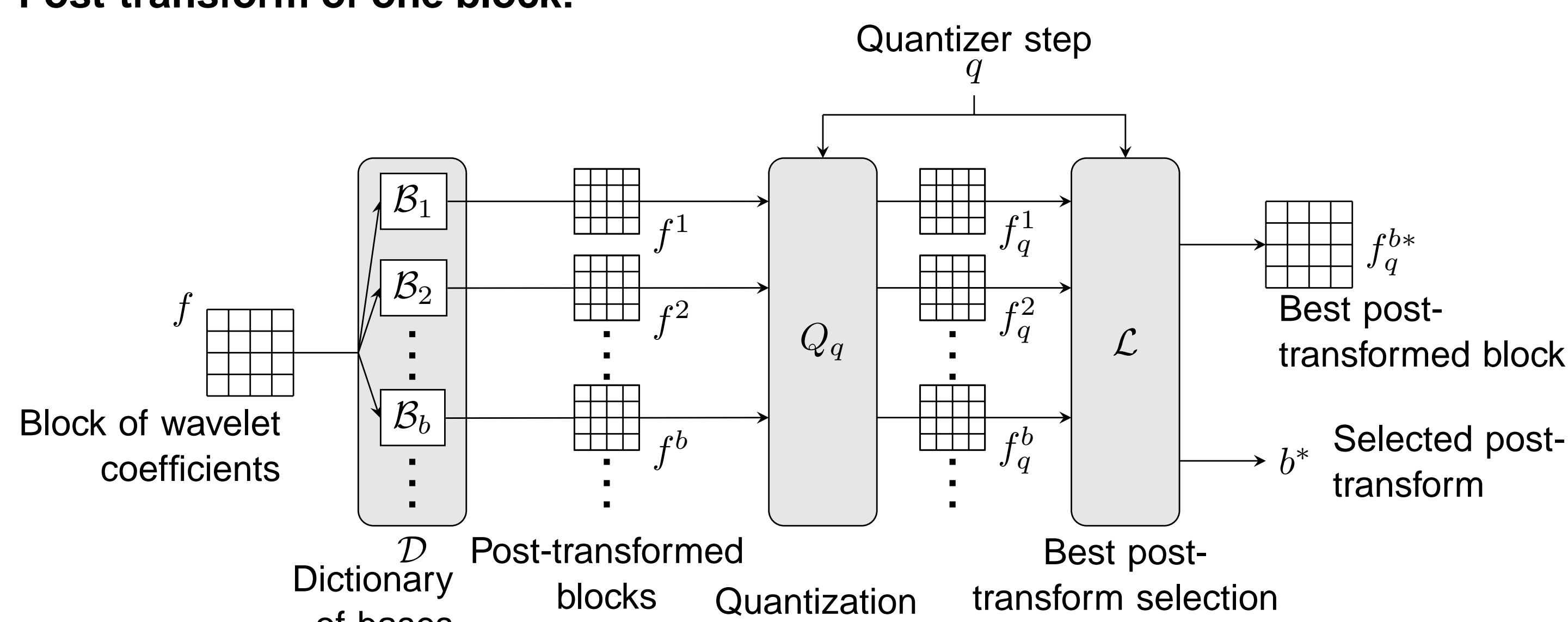
Goal: Exploit residual intra-band dependencies between wavelet coefficients.

Approach: Further transformation of blocks of wavelet coefficients using bases selected in a dictionary known both by the encoder and the decoder.

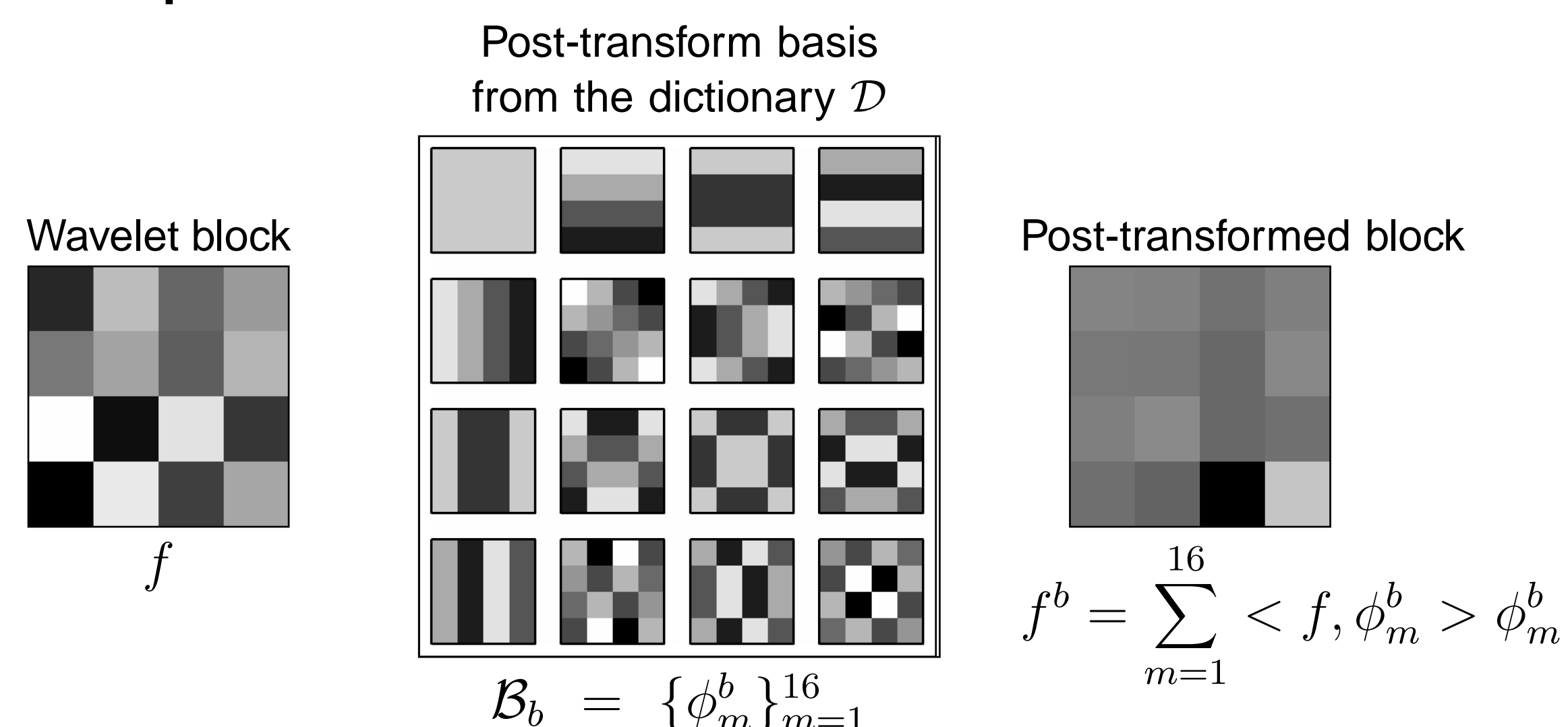
Compression scheme:



Post-transform of one block:



Example of a post-transform:



Question: What is the best post-transform basis on each block?

RATE-DISTORTION CRITERION

Goal for the compression:

Constrained problem:

Minimize the distortion $D(S)$ given the maximum target bit-rate R_T .

where S is the set of the post-transform bases selected on each block.

Unconstrained problem:

Minimize the Lagrangian rate-distortion criterion:

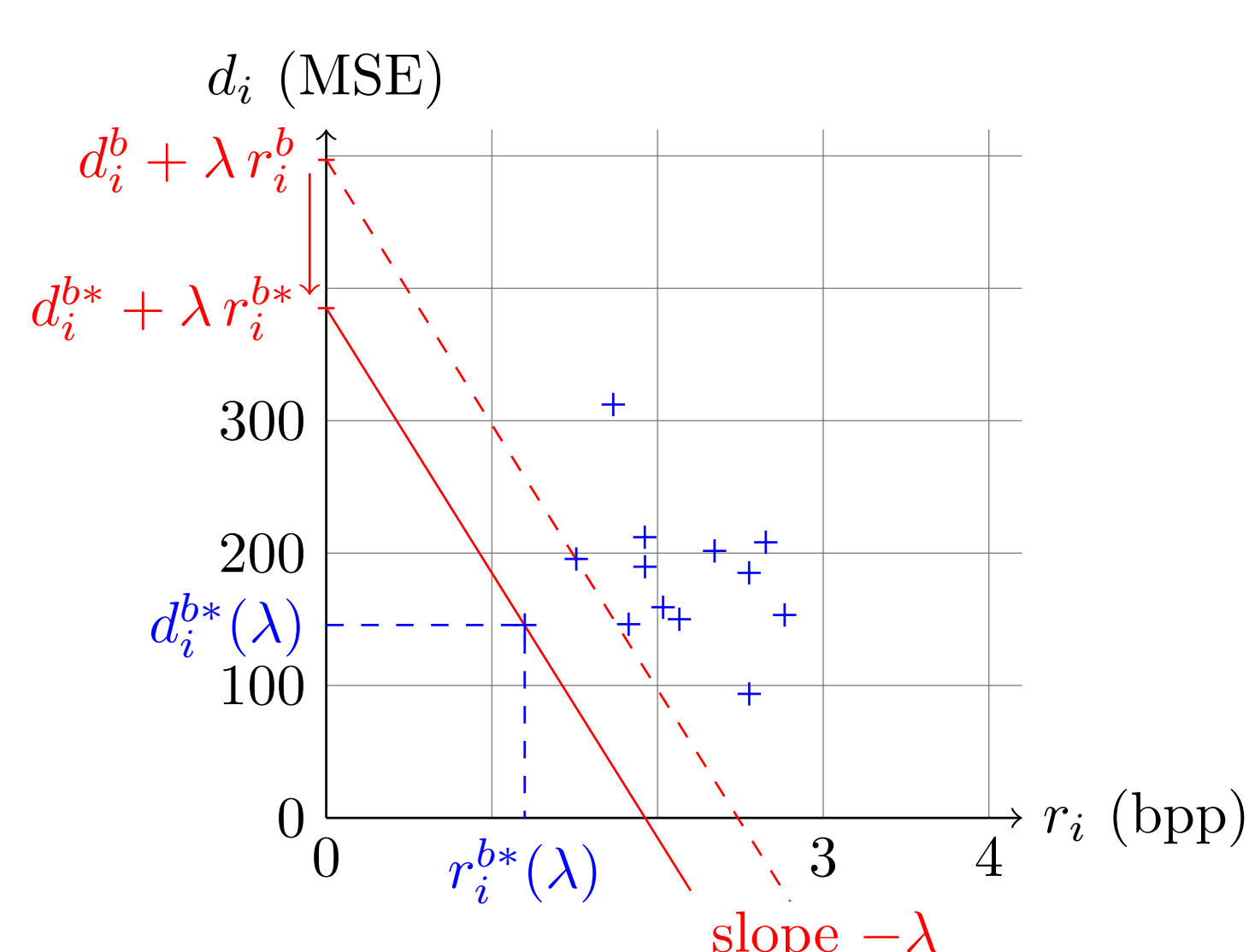
Problems:

1. How to find the best set of the post-transform bases S^* ?
2. What is the best Lagrangian multiplier λ^* ?
3. The rate R and the distortion D depend *both* on the set of the post-transform bases S and on the quantization step q !

BEST SET OF POST-TRANSFORMS

Thanks to the additive rate-distortion criterion, the best set of post-transform bases can be found by minimizing $(d_i^b + \lambda r_i^b)$ for each post-transform basis $B_b \in \mathcal{D}$ on each block of wavelet coefficient f_i .

Example for one wavelet block f_i and a dictionary \mathcal{D} of several post-transform bases B_b . The blue crosses are the (r_i^b, d_i^b) computed for the post-transformed blocks f_i^b with a fixed quantization step q .



Algorithm 1: Optimal rate-distortion point

Input: The Lagrangian multiplier λ and the rate-distortion points $\{(r_i^b, d_i^b)\}_{b,i}$ of each block f_i transformed in each basis b

Output: The optimal rate $R^*(\lambda)$ and distortion $D^*(\lambda)$

foreach block f_i **do**

// Select the representation which minimizes the Lagrangian cost

$b_i^*(\lambda) = \arg \min_{b \in [1, N_B]} (d_i^b + \lambda r_i^b)$

end

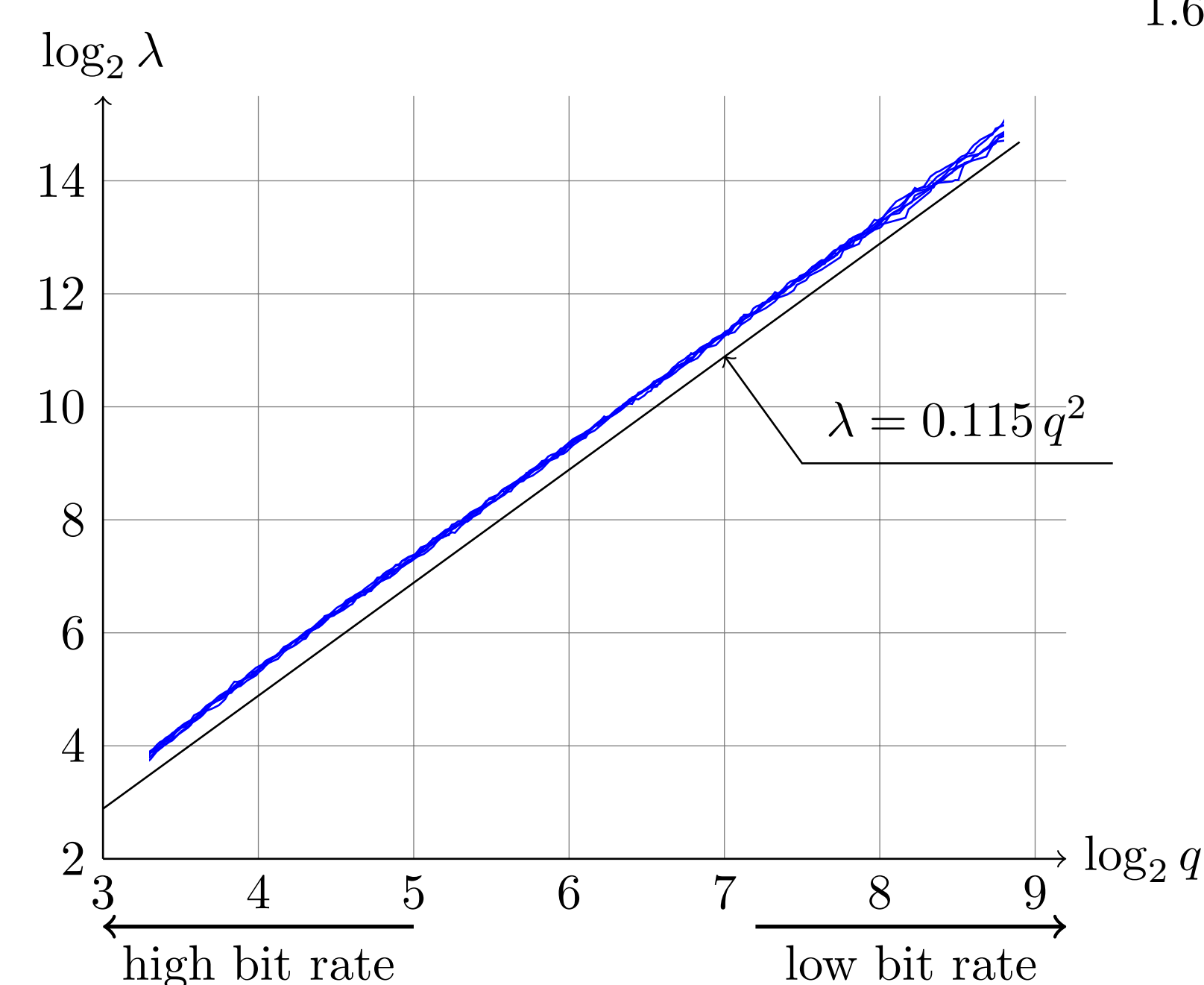
$R^*(\lambda) = \sum_i r_i^{b_i^*}(\lambda)$ and $D^*(\lambda) = \sum_i d_i^{b_i^*}(\lambda)$ and $S^*(\lambda) = \{b_i^*(\lambda)\}_i$

BEST LAGRANGIAN MULTIPLIER

Several optimal rate-distortion curves are obtained for one image by varying λ and the quantization step q .

We are looking for the lower hull of these rate-distortion curves i.e. a relation between λ and q .

$q^*(\lambda) = \arg \min_q (D^*(\lambda, q) + \lambda R^*(\lambda, q))$



Theory: (see paper)
 $\lambda = 0.115 q^2$ (black line)

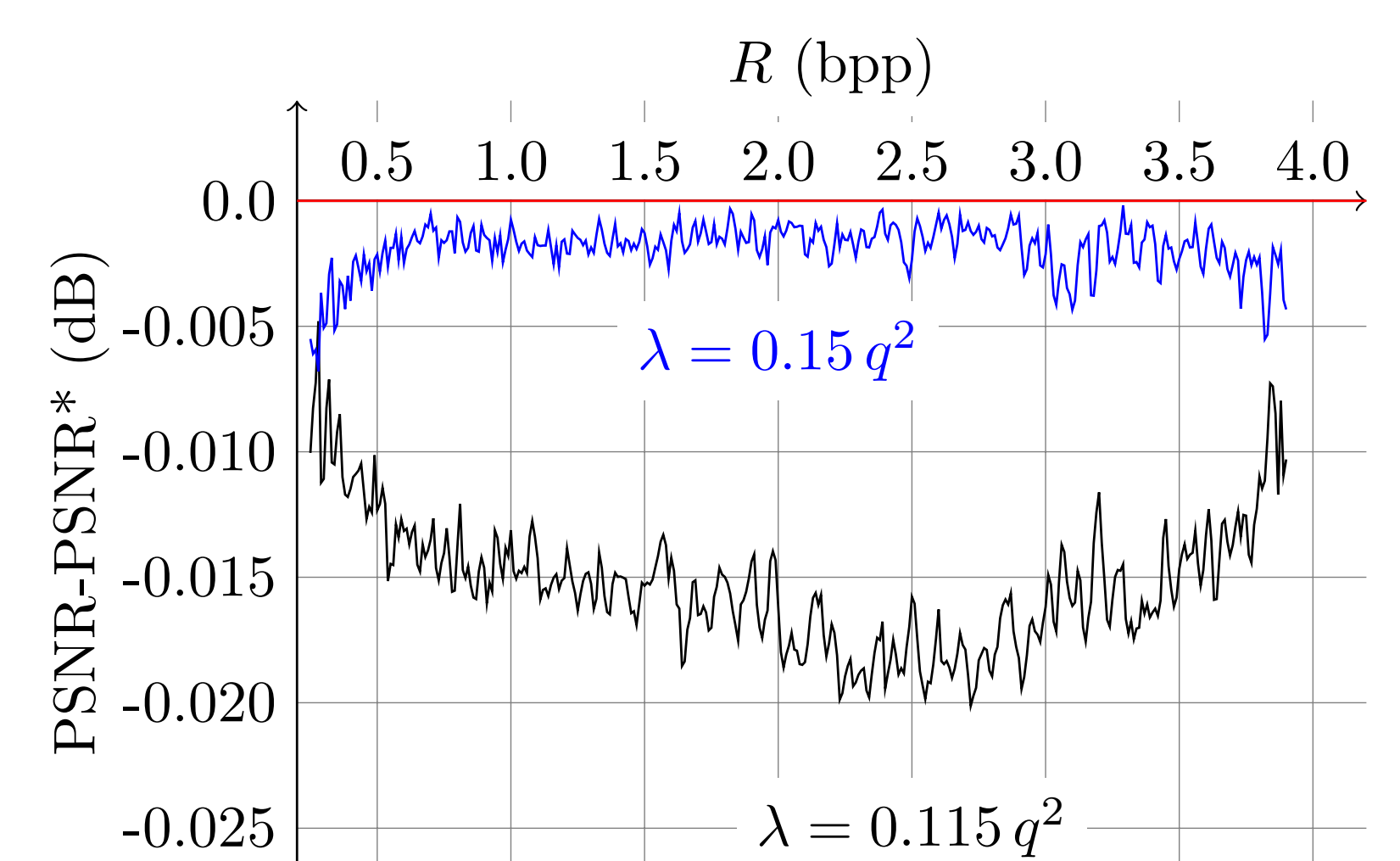
Results on a set of six Earth observation images:
 $\lambda \approx 0.15 q^2$ (blue curves)

COMPRESSION RESULTS COMPARISONS

Compression results are compared to the best compression results PSNR* obtained by a nearly exhaustive search of the best parameters (λ, q) (red line) with a second set of Earth observation images.

The blue and black curves are mean compression results obtained with the theoretical and empirical formulas of λ .

Compression results are very close to the optimal with both formulas.



CONCLUSIONS

- The best post-transforms selection amounts to the search of an optimal Lagrangian multiplier λ .
- λ depends on the quantization step q .
- λ can be computed by a nearly exhaustive search.
- Close to optimal compression results are obtained by computing the λ using a formula. This highly reduces the complexity.
- Compression results obtained with the empirical formula of λ are very close to the optimal results.

Future works: Best post-transform selection with a bit-plane encoder.

Related papers:

- X. Delaunay, M. Chabert, V. Charvillat, G. Morin and R. Ruiloba, "Satellite image compression by directional decorrelation of wavelet coefficients," in *ICASSP'08*, April 2008, Las Vegas, NV.
- X. Delaunay *et al.*, "Lossy compression by post-transforms in the wavelet domain," in *OBPDC Workshop*, June 2008, ESA ESTEC Noordwijk, The Netherlands.